PAPER CODE NO. **PHYS393**	EXAMINER: DEPARTMENT:	Dr. K. M. Hock Physics	TEL. NO. 01925-603853

JANUARY EXAMINATIONS 2013

Bachelor of Science: Year 3
Master of Physics: Year 3
Master of Physics: Year 4

STATISTICAL AND LOW TEMPERATURE PHYSICS

TIME ALLOWED: 3 hours

INSTRUCTIONS TO CANDIDATES

Answer all questions.

Question 1 carries 50% of the total marks.

Questions 2 and 3 each carry 25% of the total marks.

Answer either part (a) or part (b) of questions 2 and 3.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

The marks allotted to each part of a question are indicated in square brackets.

All symbols have their usual meanings unless otherwise stated.

Question 1.

(a) Cerium Magnesium Nitrate (CMN) is a spin ½ salt. One mole of CMN at 1 K is placed in magnetic field of 1 T.

i) What are the energies of the magnetic energy levels? **[2]**

ii) What is the name for the distribution of the spin ½ ions? State the formula for this distribution. Explain the symbols used. **[2]**

iii) State the formula for the Boltzmann factor. Calculate it for each energy level. **[2]**

iv) Starting from the formula for the distribution, explain how to find the ratio of the number of spin ½ ions in the lower level to the number in the higher level using the answers from (iii). Write down this ratio. **[2]**

v) Using this ratio, find the number of moles of spin ½ ions in each energy level. **[2]**

(b) A cubic box contains 1 mole of neon gas (relative atomic mass = 20) at 300 K.

i) State the formula for the average kinetic energy of the neon atoms. Calculate this energy. **[2]**

ii) State the formula relating energy and wavevector. Find the wavevector that corresponds to the average kinetic energy of the neon atoms. **[3]**

iii) Find the spacing between wavevectors. The side of the cubic box is 30 cm. **[2]**

iv) The total energy can be calculated by adding up the energies of all the atoms directly. Why do we still need density of states? **[3]**

(c) There is some spin ½ salt in a magnetic field. At high temperatures, there are 0.1 mole of the spin ½ ions at each of the two magnetic energy levels.

i) When temperature is lowered to 1 K, the number of the ions at the lower level increases by 50%. Find the amount of spin ½ ions at each level in moles. **[3]**

ii) Using this result, calculate the energy difference between the two energy levels. **[3]**

iii) Hence, calculate the amount heat is given out by the salt when temperature is lowered to 1 K. **[4]**

(d)

i) Why is the Faraday's law of electromagnetic induction not able to explain Meissner's effect? **[2]**

ii) The presence of a magnetic field in a macroscopic wavefunction of electrons must produce a current. Why is this able explain the Meissner's effect? **[2]**

iii) In the Meissner's effect, some magnetic field remains in the superconductor. Why is the field not completely expelled? What is the name given to the depth of penetration? **[3]**

iv) Sketch the heat capacity versus temperature graph for a superconductor, above and below the transition temperature. On the same graph, sketch the heat capacity if a magnetic field destroys the superconductivity. How do these graphs suggest the existence of an energy gap? **[3]**

(e) A body can move with zero resistance through superfluid ^{4}He.

i) What excitations are possible in superfluid ^{4}He? **[2]**

ii) Write down the dispersion relation for a particle in free space. With the help of a graph, obtain the critical velocity of a body moving through an ideal gas. **[2]**

iii) Sketch the dispersion curve of superfluid ^{4}He. Explain the main features and how to estimate the critical velocity. **[4]**

iv) Why does the body experience no resistance when its velocity is below the minimum E/p? **[2]**

Question 2. Answer **either** (a) **or** (b)

2(a)

In copper metal, each atom provides one conduction electron.

i) State the formula for the average kinetic energy of the conduction electrons, assuming that they behave like an ideal gas. Find the heat capacity of one mole of copper due to these conduction electrons. **[3]**

ii) The molar heat capacity of copper is found to be 0.6 mJ/K at 1 K. Why is this so much smaller than the answer in (i)? **[2]**

iii) If temperature is not too high, only the electrons that are just below the Fermi level are excited. Using a graph, estimate the energy interval below Fermi level from which electrons are excited. Give the answer in terms of the temperature, T. **[4]**

iv) Following on from (iii) and given that the density of states for the ideal gas is

$$g(\varepsilon) = \frac{4m\pi V}{h^3}\sqrt{2m\varepsilon}\,,$$

where V is the volume, derive an expression for the number, n, of excited electrons, in terms of Fermi energy E_F. **[4]**

v) Using a graph, explain what is the approximate distribution of these excited electrons considered in (iii) and (iv). Why do these excited electrons seem tp behave like an ideal gas? **[4]**

vi) The ideal gas formula for heat capacity is given by the answer to (i). Using this and the answer to (iv), derive an expression for the heat capacity due to these excited electrons. **[4]**

vii) Find the Fermi energy

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3},$$

where N is the number of electrons and V is the volume. (Molar volume of copper is 7.11 cm^3.)**[2]**

viii) Using the answer to (vi), find the heat capacity at 1 K. Compare with values in (i) and (ii) and comment. **[2]**

2 (b)

Dilution cooling makes use of a solution of liquid ^{3}He in liquid ^{4}He.

i) Sketch a phase diagram of the mixture. Label the axes, the lambda line and the phase separation region. **[5]**

ii) Describe qualitatively what happens when the temperature of a 50% mixture falls just below the phase separation curve. **[5]**

iii) Describe qualitatively what happens when the temperature of this mixture reaches a few milliKelvin. **[3]**

iv) Sketch a diagram to explain qualitatively how the ^{3}He could be removed from the bottom layer. **[6]**

v) Why is it necessary to remove ^{3}He from the bottom layer? How does this affect the amount of ^{3}He in the top layer? What must be done to maintain the same amount of ^{3}He in the top layer? **[6]**

Question 3. Answer **either** (a) **or** (b)

(a)

i) Discuss the main features of Bose-Einstein condensate. Suggest why it is a good candidate for explaining superfluidity. **[5]**

ii) The chemical potential μ of a boson gas changes as temperature falls to 0 K. Explain how it changes, using the Bose-Einstein distribution graph. **[5]**

iii) Write down the formula for the number of bosons N in terms of density of states $g(\varepsilon)$, where ε is energy of a particle. When temperature T falls below a certain value, μ has to be set to zero. Sketch the graph of N against T and explain the main features. **[5]**

iv) When μ is zero, the result of the integral in (iii) is

$$N_{ex} = 2.612V\left(\frac{2\pi m k_B T}{h^2}\right)^{3/2},$$

where V is the volume that contains the N particles. Explain how to find the condensation temperature, T_{BE}. Find T_{BE} for liquid ^{4}He (molar volume 27.58 cm^3). **[5]**

v) The total energy U below T_{BE} is given by

$$U = 0.7704 k_B N \frac{T^{5/2}}{T_{BE}^{\ 3/2}}.$$

Derive the heat capacity, C, and find it for one mole of ^{4}He at T_{BE}. Sketch the graph of C versus T above and below T_{BE}. Give the answer in units of the gas constant R. Explain the main features.

[5]

3 (b)

In a superconducting metal, the electrons form Cooper pairs. A Cooper pair is a pair of electrons which attract each other.

i) This means that some energy is needed to separate them. Describe one experimental result which suggests the existence of such a binding energy. **[5]**

ii) The superconducting property depends on vibration of the atoms. Describe one experiment which demonstrates this. **[5]**

iii) With the help of a picture, explain qualitatively how movements of the atoms could cause two electrons to attract. **[5]**

iv) The binding energy in the Cooper pair is expected to be much smaller than the kinetic energy of the electrons. Using the Fermi level, explain why the two electrons do not separate from each other. **[5]**

v) A macroscopic wavefunction has been used to explain the Meissner's effect. What is wrong with this idea? How does forming of Cooper pairs help to solve this problem? **[5]**

CONSTANTS

Speed of light in vacuum	c	=	3.00×10^{8} ms^{-1}
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$ Hm^{-1}
		=	$4\pi \times 10^{-7}$ VsA^{-1}m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12} Fm^{-1}
		=	8.85×10^{-12} AsV^{-1}m^{-1}
Elementary charge	e	=	1.60×10^{-19} C
Planck constant	h	=	6.63×10^{-34} Js
	$h/2\pi = \hbar$	=	1.05×10^{-34} Js
Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	=	1.38×10^{-23} JK^{-1}
Gas constant	R	=	8.31 JK^{-1}mol^{-1}
Unified atomic mass constant	m_u	=	1.66×10^{-27} kg
		=	931.5 MeVc^{-2}
Electron mass	m_e	=	9.11×10^{-31} kg
Proton mass	m_p	=	1.67×10^{-27} kg
Gravitational constant	G	=	6.67×10^{-11} Nm^{2}kg^{-2}
Acceleration due to gravity	g	=	9.8 ms^{-2}
Bohr magneton	μ_B	=	9.27×10^{-24} JT^{-1}